# VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD 

 B.E. (CBCS) III-Semester Backlog (Old) Examinations, December-2018 Engineering Mathematics-IIITime: $\mathbf{3}$ hours
Max. Marks: 70
Note: Answer ALL questions in Part-A and any FIVE questions from Part-B

## Part-A (10 $\times 2=20$ Marks)

1. State the conditions under which a given function can be expanded in Fourier series.
2. Find the value of $a_{0}$ in the Fourier expansion of the function $f(x)=$ $\{1+t,-1 \leq t \leq 0$ $\{1-t, 0 \leq t \leq 1$
3. Deduce the Partial differential equation by elimination of the arbitrary constants $a$ and $b$ from the equation $z=a x e^{y}+\frac{1}{2} a^{2} e^{2 y}+b$
4. Explore the solution of the partial differential equation $p-q=\frac{z}{x+y}$
5. Establish the relation between the operators (i) $\Delta$ and $E$ (ii) $\nabla$ and $E^{-1}$
6. By froosing an appropriate Interpolation formula, construct a second degree polynomial for the following data: $(1,3),(2,5),(3,10)$.
7. Wite any four properties of the Normal Distribution.
8. Prove that $E(X+Y)=E(X)+E(Y)$
9. The equations of two regression lines obtained in a correlation analysis are $3 x+2 y=26$ and $6 x+y=31$. Find (i) the correlation coefficient $r$, and (ii) The mean values of $x$ and $y$
10. Show that the limits of correlation coefficient $r$ are $-1 \leq r \leq+1$

## Part-B $(5 \times 10=50$ Marks $)$

11. a) Expand the function $f(x)=\left(\frac{\pi-x}{2}\right)^{2}$ in $0 \leq x \leq 2 \pi$
b) Find the half range sine and cosine series of $f(x)=x$, in $0<x<2$
12. a) Solve the differential equation $z^{2}\left(p^{2} z^{2}+q^{2}\right)=1$
b) Solve the partial differential equation $\frac{\partial u}{\partial t}=\alpha^{2} \frac{\partial^{2} u}{\partial x^{2}}$ for the conduction of heat along the rod without radiation, subject to the following conductions
(i) $u$ is not infinite for $t \rightarrow \infty$
(ii) $\frac{\partial u}{\partial x}=0$ for $x=0$ and $x=l$
(iii) $u(x, 0)=l x-x^{2}$ for $t=0$, between $x=0 \& x=l$
13. a) A body is moving with velocity $v$ at any given time $t$ and satisfies the following data

| $t$ | 0 | 1 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| $v$ | 21 | 15 | 12 | 10 |

Obtain the distance travelled in 4 seconds and acceleration at the end of 4 seconds.
b) Obtain the approximate value of $y$ at $x=1$ in steps of 0.2 by Euler's method given

$$
\begin{equation*}
\frac{d y}{d x}=x y \text { and } y(0)=2, \tag{5}
\end{equation*}
$$

14. a) If the p.d.f. $f(x)=k(x+3)$ in $(2,8)$, determine the value of $k$ and
(i) $P(3<x<5)$, (ii) $P(x \geq 4)$
b) Derive the mean and Variance of Normal distribution.
15. a) If $\theta$ is the angle between the two regression lines in the case of two variables $x$ and $y$,

Show that $\tan \theta=\left(\frac{1-r^{2}}{r}\right) \frac{\sigma_{x} \sigma_{y}}{\sigma^{2} x+\sigma^{2} y}$, and interpret the result for different values of $\theta$.
b) Calculate the coefficient of correlation and obtain the least square regression lines for the following data:

| $x$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 2 | 5 | 3 | 8 | 7 |

16. a) Find the Fourier series for the function $f(x)=\left\{\begin{array}{cc}x, & 0<x<1 \\ 2-x, & 1<x<2\end{array}\right.$
b) Solve the partial differential equation $2(z+x p+y q)=y p^{2}$ by Charpit's method.
17. Answer any two of the following:
a) Determine $y^{\prime}(0)$ and $y^{\prime \prime}(0)$ for the following data:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | 4 | 8 | 15 | 7 | 6 | 2 |

b) The marks X obtained in mathematics by 1000 students is normally distributed with mean $78 \%$ and standard deviation $11 \%$. Determine (i) How many students got marks above $90 \%$ ? (ii) What was the highest mark obtained by the lowest $10 \%$ of students?
c) The following table gives the number of aircraft accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week.

| Day | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of accidents | 14 | 16 | 8 | 12 | 11 | 9 | 14 |

